Ecological Discount Rate and Precautionary Principle

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Motivations

Two related questions:

What is the relevant discount rate for environmental issues?
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- Main question for cost-benefit analysis (Stern Review).
- Long term horizon.
- Theoretical point: substitutability between consumption good and the environment.
- Output: Micro-foundation of low discount rates.
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  - Long term horizon.
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Precautionary principle

Underlying ideas

- Weak PP
- Strong PP
Motivations

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- Main question for cost-benefit analysis (Stern Review).
- Long term horizon.
- Theoretical point: substitutability between consumption good and the environment.
- Output: Micro-foundation of low discount rates.

Precautionary principle
- How to justify a precautionary principle?
- What risk really is? Ignorance of events or ignorance of their impact on welfare?
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Introduction

1. The model
   - Setup of the model
   - Discount rates
   - Optimization problem

2. Results

3. Precautionary principle
In a nutshell

A two-good model

- A consumption good (with no limitation): $x$
- An environmental good (available in finite quantity): $y$
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A four-parameter model
A two-good model

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- $\delta$: pure rate of time-preference.
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A four-parameter model
- $\delta$: pure rate of time-preference.
- $r$: (financial) interest rate.
A two-good model

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- $\delta$: pure rate of time-preference.
- $r$: (financial) interest rate.
- $\sigma$: elasticity of substitution between the two goods.
In a nutshell

A two-good model
- A consumption good (with no limitation): $x$
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A four-parameter model
- $\delta$: pure rate of time-preference.
- $r$: (financial) interest rate.
- $\sigma$: elasticity of substitution between the two goods.
- $\sigma'$: (inverse) inter-temporal elasticity of substitution.
Some more details

### CES ordinal utility function

\[
v(x_t, y_t) = \left[ \frac{1}{\sigma} x_t^{\sigma-1} + y_t^{\sigma-1} \right]^{\frac{\sigma}{\sigma-1}}
\]
Some more details

**CES ordinal utility function**

\[ v(x_t, y_t) = \left[ x_t^{\frac{\sigma-1}{\sigma}} + y_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \]

**Cardinal utility**

\[ V(x_t, y_t) = \frac{1}{1 - \sigma'} v(x_t, y_t)^{1-\sigma'} \]
Some more details

**CES ordinal utility function**

\[ v(x_t, y_t) = \left[ \frac{\sigma - 1}{\sigma} x_t^\sigma + \frac{\sigma - 1}{\sigma} y_t^\sigma \right]^{\frac{\sigma}{\sigma - 1}} \]

**Cardinal utility**

\[ V(x_t, y_t) = \frac{1}{1 - \sigma'} v(x_t, y_t)^{1 - \sigma'} \]

**Welfare function**

\[ \sum_{t=0}^{\infty} e^{-\delta t} V(x_t, y_t) \]

\( x_t \) has to be paid for. \( y_t \) is free but available in limited quantity.
Notion of discount rates I

We consider a reference trajectory \((x_t, \bar{y})\)

**Implicit discount rate (private good)**

The implicit discount rate for private good between periods \(t\) and \(t + 1\), is \(r_t^*\) such that Euler condition is satisfied:

\[
e^{-r_t^*} = e^{-\delta} \frac{\partial_1 V(x_{t+1}, \bar{y})}{\partial_1 V(x_t, \bar{y})}
\]

We introduce:

\[
R^*(T) = \frac{1}{T} \sum_{t=0}^{T-1} r_t^*
\]
Similarly, the ecological implicit discount rate between two consecutive periods is $\beta_t^*$ defined by:

$$e^{-\beta_t^*} = e^{-\delta} \frac{\partial^2 V(x_{t+1}, \bar{y})}{\partial^2 V(x_t, \bar{y})}$$

and we denote:

$$B^*(T) = \frac{1}{T} \sum_{t=0}^{T-1} \beta_t^*$$

Remark: A cross discount rate will be dealt with further on.
The link between $\beta_t^*$ and $r_t^*$ depends on consumption growth. Easy computations lead to:

$$\beta_t^* = r_t^* - g_t^*/\sigma$$

- If the interest rate is given ($r$), the ecological discount rate is lesser than $r$.
- The more you grow, the more you “need” environmental goods when $t$ is large.
- We see the role of substitutability between the two goods ($\sigma$).

Now, we need to make the growth path endogenous...
A representative agent or a social planner has the following optimization problem:
A representative agent or a social planner has the following optimization problem:

$$\sum_{t=0}^{\infty} e^{-\delta t} V(x_t, y_t)$$

subject to:

$$\alpha_{t+1} = e^r (\alpha_t - x_t) \quad y_t \leq \bar{y}$$

where $\alpha$ stands for wealth and $\alpha_0$ is given.

We have to...

- find the consumption path...
- deduce the growth path...
- conclude on the ecological discount rates:
  - Asymptotic results.
  - Finite-time results.
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Asymptotic results

Non-asymptotic results

Poor/Rich countries

Precautionary principle

Underlying ideas

Weak PP

Strong PP
Asymptotic results
Asymptotic results

Asymptotic growth rate

- $\sigma < 1$: $g^* = \sigma (r - \delta)$
- $\sigma > 1$: $g^* = \frac{r - \delta}{\sigma'}$
- $\sigma = 1$: specific
Asymptotic results

Asymptotic growth rate

- $\sigma < 1$: $g^*_\infty = \sigma(r - \delta)$
- $\sigma > 1$: $g^*_\infty = \frac{r-\delta}{\sigma'}$
- $\sigma = 1$: specific

Asymptotic ecological discount rate

- $\sigma < 1$: $B^*_\infty = \delta$
- $\sigma > 1$: $B^*_\infty = (1 - \frac{1}{\sigma\sigma'})r + \frac{1}{\sigma\sigma'}\delta$
- $\sigma = 1$: specific
Comments

- Elasticity of substitution between the two goods is the essential parameter.
- $\sigma > 1$: the asymptotic growth is not affected by the consideration of environmental issues
- $\sigma < 1$: In the long run, environmental issues are primordial and $B^* = \delta$!
- Generalizations are available: Formulae can be derived when $\bar{y}$ decreases with time.
- The discontinuity may seem cumbersome, we will see that it is not a problem.
Finite-time results

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∀T < ∞, σ ̂ → B∗(T; σ) is continuous.
The discontinuity is just a problem of double limit.

Shape of the yield curve
If σ > 1 (resp. σ < 1) and σ < 1 then T ̂ → B∗(T) is decreasing (resp. increasing) and converges towards δ.
If σ > 1 (resp. σ < 1) and σ > 1 then T ̂ → B∗(T) is increasing (resp. decreasing) and converges towards \left(\frac{1}{\sigma} - \frac{1}{\sigma} \right)^{-1} (r + 1).
The most relevant case being σ > 1, this result advocates for low discount rates even in finite horizon.
Finite-time results

**Continuity**

\[ \forall T < \infty, \sigma \mapsto B^*(T; \sigma) \text{ is continuous.} \]

The discontinuity is just a problem of double limit.
Finite-time results

Continuity

\[ \forall T < \infty, \sigma \mapsto B^*(T; \sigma) \text{ is continuous.} \]

The discontinuity is just a problem of double limit.

Shape of the yield curve

- If \( \sigma \sigma' > 1 \) (resp. \( \sigma \sigma' < 1 \)) and \( \sigma < 1 \) then \( T \mapsto B^*(T) \) is decreasing (resp. increasing) and converges towards \( \delta \).
- If \( \sigma \sigma' > 1 \) (resp. \( \sigma \sigma' < 1 \)) and \( \sigma > 1 \) then \( T \mapsto B^*(T) \) is increasing (resp. decreasing) and converges towards

\[
\left(1 - \frac{1}{\sigma \sigma'}\right) r + \frac{1}{\sigma \sigma'} \delta
\]

The most relevant case being \( \sigma \sigma' > 1 \), this result advocates for low discount rates even in finite horizon.
Simulations

Figure: Yield curve example ($\sigma = 0.8$, $\sigma' = 1.5$, $r = 2\%$, $\delta = 0.1\%$)
Simulations

Figure: Yield curve example ($\sigma = 1.2$, $\sigma' = 1.5$, $r = 2\%$, $\delta = 0.1\%$)
Bounds for perpetuity bonds I

- Yield curves contain all the information.
- It may be interesting to have a unique figure.
- Environmental perpetuity:

\[ m = \sum_{T=1}^{\infty} \exp(-B^*(T)T) \]

- Interpretation: if I’m ready to pay \( x \) to avoid an environmental damage for this year, I’m ready to pay \( mx \) to avoid the same damage forever.
Bounds for perpetuity bonds II

**Lower bounds for** $m$

Let’s assume $\sigma \sigma' > 1$ (we do not assume anything on $\sigma$ itself).

Then:

$$m > \frac{1}{r(1 - \frac{1}{\sigma \sigma' }) + \delta \frac{1}{\sigma \sigma'}}$$

**Remark:** if $\sigma \sigma' < 1$, then $m > \frac{1}{\delta}$.

**Example:** ($r = 3\%$, $\delta = 1\%$ and $\sigma' = 1.5$)

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>$\sigma_h = 1.2$</td>
<td>$\sigma_l = 0.8$</td>
</tr>
<tr>
<td>Theoretic inferior bound for $m$</td>
<td>52.94</td>
<td>75</td>
</tr>
<tr>
<td>Actual $m$</td>
<td>61.49</td>
<td>86.68</td>
</tr>
</tbody>
</table>
Investment in the environment

- Concept of ecological “return”.
- Unitary initial investment $\Rightarrow$ improvement $\Phi$ of the “environmental” quality at time $T$.
- $\Phi = e^{\Omega^*(T)T}$ is just socially profitable $\iff e^{-B^*(T)T} e^{\Omega^*(T)T} (\frac{x_0}{y})^{1/\sigma} = 1$:
  $\Omega^*(T) = B^*(T) - (\frac{1}{T})(1/\sigma) \ln(\frac{x_0}{y})$
- Same asymptotic behavior. Different “yield” curves.
- It encourages subsidies (Mediterranean Sea).
Figure: Yield Curve for $\Omega$ ($\sigma = 0.8$, $\sigma' = 1.5$, $r = 2\%$, $\delta = 0.1\%$, $\bar{y} \ll x_0^*$)
Figure: Yield Curve for $\Omega$ ($\sigma = 0.8$, $\sigma' = 1.5$, $r = 2\%$, $\delta = 0.1\%$, $\bar{y} \gg x_0^*$)
Figure: Yield Curve for $\Omega$ ($\sigma = 0.8$, $\sigma' = 1.5$, $r = 2\%$, $\delta = 0.1\%$, \(\bar{y} \sim x_0^*\))
Introduction

The model

Results

Precautionary principle
  - Underlying ideas
  - Weak PP
  - Strong PP
What is risk?

- Large literature on fat tails, extreme events, ...
- In the evaluation of a risk by $\mathbb{E}[u(X)]$, the risk is usually linked to $X$.
- For climate change, we argue that the risk is twofold:
  - Classic risk: extent or amplitude of the consequences of climate change.
    ⇒ $X$
  - Welfare risk: Will environment turn out to be really important for us, once a catastrophe occurred?
    ⇒ $u$

$\sigma$ is basically unknown before an event concerning environment.
We consider a deterministic (to simplify) event at time $\tau$.

$\sigma$ can be $\sigma_l < 1$ or $\sigma_h > 1$ and we discover its value at time $\tau$.

Before time $\tau$, we attribute a priori beliefs $p$ and $1 - p$.

New optimization problem

$$\sum_{t=0}^{\tau-1} e^{-\delta t} [pV(\sigma_l; x_t, \bar{y}) + (1 - p)V(\sigma_h; x_t, \bar{y})] + p\mathcal{U}(\alpha_\tau, \sigma_l) + (1 - p)\mathcal{U}(\alpha_\tau, \sigma_h)$$

where $\mathcal{U}(\alpha, \sigma) = \text{Max}_{(x_t, y_t)_{t \geq \tau}} \sum_{t=\tau}^{\infty} e^{-\delta t} V(\sigma; x_t, y_t)$

(Same constraints)
Asymptotically, the smallest possible rate applies. More formally:

**Weak precautionary principle**

Let’s assume $p \in (0; 1)$ and $\sigma_h \sigma' > 1$, then $B_\infty^*$ does not depend on $p$:

$$B_\infty^* = \delta$$

This is not surprising but it doesn’t say anything on $B^*(T)$ for finite $T$’s.
\( p \mapsto m(p) \)

We can go back to \( m \) that gathers information on the \( B(T) \)'s in a meaningful way.
We have the following result for \( \sigma_h\sigma' > 1 \) and \( \sigma_l\sigma' > 1 \) (to simplify formulae):

**Strong precautionary principle**

Let’s introduce 
\[
a(h) = r(1 - \frac{1}{\sigma_h\sigma'}) + \delta \frac{1}{\sigma_h\sigma'}
\]
\[
a(l) = r(1 - \frac{1}{\sigma_l\sigma'}) + \delta \frac{1}{\sigma_l\sigma'}.
\]

We have:

\[
m > e^{-B^*(\tau)\tau} \left[ \frac{pN^*(\tau)\frac{1}{a(l)}}{pN^*(\tau) + (1-p)} + (1-p)\frac{1}{a(h)} \right]
\]

where \( N^*(\tau) \) grows exponentially with \( \tau \).

⇒ Concavity property, well illustrated by simulations
Simulations $\Rightarrow$ Strong Precautionary Principle

**Figure:** Value of $m$. $\sigma \in \{0.8; 1.2\}$ is revealed at $\tau = 100$. $r = 3\%, \delta = 1\%, \sigma' = 1.5$